Sheet 2

1 Determine the volume V of a region defined in a cylindrical coordinate system as $1m \leq r \leq 2m$, $0 \leq \phi \leq \frac{\pi}{3}$ rad , and $0 \leq z \leq 1\ m$ by integration . Check your result without performing the integration.

$$\left[\boldsymbol{V = \frac{\pi}{2}\ m^3}\right]$$

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2 Determine the area S of a surface in a spherical coordinate system as $r = 2\ m$ and $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{3}$ rad .

$$[\boldsymbol{A = 5.205\ m^2}]$$

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3 Use the spherical coordinate system to find the area of the strip $\alpha \leq \theta \leq \beta$ on the spherical shell of $r = a$ show this strip by sketching. What result when $\alpha = 0$ and $\beta = \pi$?

$$\left[\begin{array}{c}\boldsymbol{A = 2\pi a^2[cos\,\alpha - cos\,\beta\,]} \\ \boldsymbol{A = 4\pi a^2}\end{array}\right]$$

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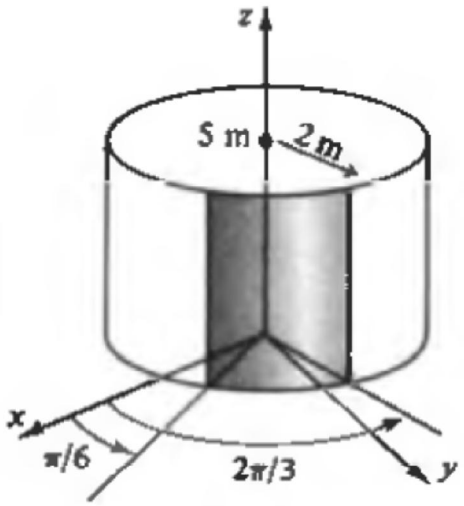
4 Use the cylindrical coordinate system to find the area of the curved surface of a right circular cylinder where: $r = 2m$, $h = 5m$ and $30° \leq \phi \leq 120°$ as shown in fig



$$[\boldsymbol{A = 5\pi\ m^2}]$$

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5 Given the point $P\ (5\text{m}, 60°, 2\text{m})$ and $Q\ (2\text{m}, 110°, -1\text{m})$

(a) Find the distance R_{PQ}

(b) Give a unit vector in Cartesian coordinates at P that is directed towards Q

(c) Give a unit vector in cylindrical coordinates at P that is directed towards Q

$$\begin{bmatrix} \boldsymbol{R_{PQ} = 5.014\ m} \\ \boldsymbol{\bar{a}_{PQ} = -0.635\bar{a}_x - 0.489\bar{a}_y - 0.598\bar{a}_z} \\ \boldsymbol{At\ point\ P\ , \bar{a}_{PQ_{cyl}} = -0.741\ \bar{a}_\rho + 0.306\ \bar{a}_\phi - 0.598\ \bar{a}_z} \end{bmatrix}$$

6

(a) Find $\bar{a}_x$ in the spherical components at $P\ (3, -4, 5)$

(b) Find $\bar{a}_\theta$ in Cartesian components at P

$$\begin{bmatrix} \boldsymbol{\bar{a}_x = 0.424\ \bar{a}_r + 0.424\ \bar{a}_\theta + 0.8\ \bar{a}_\phi} \\ \boldsymbol{\bar{a}_\theta = 0.424\ \bar{a}_x - 0.565\ \bar{a}_y - 0.707\ \bar{a}_z} \end{bmatrix}$$

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7 A closed surface is defined in spherical coordinates by $3 \le r \le 5$,

$0.1\pi \le \theta \le 0.3\pi$, $1.2\pi \le \phi \le 1.6\pi$

(a) Find the volume enclosed.

(b) Find the total surface area.

$$\begin{bmatrix} \boldsymbol{V = 14.912} \\ \textbf{Total Area} = \boldsymbol{36.8125} \end{bmatrix}$$

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8 Transform $\bar{A} = y\bar{a}_x + x\bar{a}_y + \dfrac{x^2}{\sqrt{x^2 + y^2}}\bar{a}_z$ to cylindrical coordinates.

$$\left[\boldsymbol{\bar{A}_{cyl} = \rho \sin 2\phi\ \bar{a}_\rho + \rho \cos 2\phi\ \bar{a}_\phi + \rho \cos^2 \phi\ \bar{a}_z}\right]$$